

**HOME ASSIGNMENT (2024 Batch)**  
**MA/MS IN MATHEMATICS**  
**(FOURTH SEMESTER)**  
**CENTRE FOR DISTANCE AND ONLINE EDUCATION**  
**DIBRUGARH UNIVERSITY**  
*(Full Marks 20 for each course)*

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**Course: MATH-401 (Functional Analysis)**

Assignment 1 (10)

(i)

Define a **Banach space**. Show that the space  $\ell^p$  (for  $1 \leq p < \infty$ ) with the norm

$$\|x\|_p = \left( \sum_{k=1}^{\infty} |x_k|^p \right)^{1/p}$$

is a normed linear space.

Assignment 2 (5+5)

- (i) Define **self-adjoint operators** and **unitary operators** on Hilbert spaces. Give one example of each.
- (ii) State and explain the **Projection Theorem** in a Hilbert space. Illustrate with an example in  $\mathbb{R}^3$ .

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**Course: MATH-402 (Computer programme)**

Assignment 1 (5+5)

- (i) Explain with examples the difference between the **while loop** and the **do-while loop** in C.
- (ii) Draw a **flowchart** and write an **algorithm** to find the largest of three given numbers.

Assignment 2 (5+5)

- (i) Write a C program to **check whether a given number is prime or not** using decision-making statements.
- (ii) Write a C function to compute the **factorial of a number** using recursion.

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**Course: MATH-403(A) (Number Theory)**

Assignment 1 (10)

- (i) State and prove **Euler's Criterion** for quadratic residues. Use it to determine whether 7 is a quadratic residue modulo 19.

Assignment 2:

- (i) (10)

Find all integer solutions of the **Diophantine equation**:

$$12x + 18y = 30.$$

**Course: MATH-404(A) (Graph Theory)**

Assignment 1 (10)

- (i) Define a graph. Explain the difference between finite and infinite graphs. Give examples of isolated, pendant, and null vertices.

Assignment 2 (10)

- (i) Discuss methods to find the shortest path between all pairs of vertices. Illustrate with an example.
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**Course: MATH-403(B) (Abstract Algebra)**

Assignment 1: (5+5)

- (i) Define an  $R$ -homomorphism between modules. Discuss kernel, image, and quotient modules with examples.  
(ii) Define a splitting field. Give an example and explain its construction.

Assignment 2 (10)

- (i). Discuss the Fundamental Theorem of Algebra. Explain its connection with Galois theory.

**Course: MATH-404(B) (Operator Theory)**

Assignment 1 (5+5)

- (i) Define Banach algebra. Discuss the basic spectral properties of bounded linear operators in Banach algebras.  
(ii) Describe the impact of compactness on the resolvent set of a linear operator.

Assignment 2 (5+5)

- (i) Prove that the spectrum of a bounded self-adjoint operator lies in the real line.  
(ii) Define unbounded linear operators in Hilbert spaces. Explain the concept of Hilbert adjoint operators.
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**Course: MATH-403(C) (Magnetohydrodynamics)**

Assignment 1: (10)

- (i) Discuss the electrical properties of a conducting fluid. Explain the roles of the electric field, magnetic field, and Lorentz force in magnetohydrodynamics.

Assignment 2: (10)

- (i) Discuss Poiseuille-type flow and Couette-type flow in the context of linear MHD.

**Course: MATH-404(C) (Nonlinear Dynamical System)**

Assignment 1: (10)

- (i) Write short note on Regular Point, Double point, singular point, cusp, bifurcation of a system.

Assignment 2: (10)

- (i) Explain Lyapunov's method for stability of equilibrium points.